Fachbereich Mathematik und Informatik
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Differentialgeometrie II SS 2019

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Tutorial Sheet 1

Exercise 1 (real line with two origins). Let $O_\pm = (0, \pm 1) \in \mathbb{R}^2$ and consider

$$M = \{(x, y) \in \mathbb{R}^2 \backslash \{0\} : y = 0\} \cup \{O_+, O_-\} = U_+ \cup U_-,$$

where $U_\pm = M \backslash \{O_\mp\}$. Define mappings $\varphi_\pm : U_\pm \to \mathbb{R}$ such that

$$\varphi_\pm(x, y) = \begin{cases} x, & x \neq O_\pm \\ 0, & \text{otherwise.} \end{cases}$$

Show that $\{\varphi_+, \varphi_-\}$ constitutes a smooth 1-dimensional atlas, yet the Hausdorff axiom is violated (cf. Figure 1).

4 marks

Exercise 2. Let V be an $\mathbb{R}$-vector space and define the mapping

$$\begin{aligned} \text{𝄞} &: V \times V \to TV \\ (p, v) &\mapsto \text{𝄞}_p v := \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} (p + tv). \end{aligned}$$

Verify that 𝄞 is a diffeomorphism and for each $p \in V$, $\text{𝄞}_p : V \to T_pV$ is an isomorphism.

3 marks

Exercise 3. Let $f : Q_f \subset \mathbb{R} \times M \to M$ be a local flow and define the vector field $X_f : M \to TM$ such that for $p \in M$,

$$X_f(p) = \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} f_t(p).$$

Let $\phi : Q \to M$ denote the local flow generated by X_f. Show that $Q_f \subset Q$ and $\phi|_{Q_f} \equiv f$.

3 marks

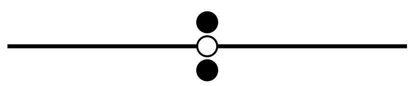

Figure 1: The real line with two origins

To be handed in by 10 a.m. on 03/05/2019.