Fachbereich Mathematik und Informatik
Freie Universität Berlin

Einführung in die Differentialgeometrie WS 2018/2019

Klaus Ecker

Sheet 1

Problem 1. Let $S^n = \{x \in \mathbf{R}^{n+1}, |x|^2 = 1\}$. Let $p \in S^n$ and $v \in \mathbf{R}^{n+1}$ satisfy the relation $p \cdot v = 0$ and assume $v \neq 0$. Find an explicit smooth curve $\alpha : I \to S^n$ which satisfies $\alpha(0) = p$ and $\dot{\alpha}(0) = v$.

3 points

Problem 2. Let C be a curve in the upper half-plane of $\mathbf{R^2}$, i.e. $C = f^{-1}(0)$ for a smooth function $f : U \to \mathbf{R}$ with $U = \{(t,r),\, r > 0,\, t \in \mathbf{R}\}$ and $\nabla f(t,r) \neq 0$ for all $(t,r) \in C$. Let $V \subset \mathbf{R}^{n+1}$ be the open set given by $V = \{x = (x_1, \ldots, x_{n+1}) \in \mathbf{R}^{n+1},\, \sqrt{x_1^2 + \ldots + x_n^2} > 0,\, x_{n+1} \in \mathbf{R}\}$ and $g : V \to \mathbf{R}$ defined by $g(x_1, \ldots, x_{n+1}) = f(x_{n+1}, \sqrt{x_1^2 + \ldots + x_n^2})$. Let $S = g^{-1}(0)$. (For $n = 2$, S is the set which one obtains by rotating the curve C about the x_3 - axis in $\mathbf{R}^3$.).

Show: S is an n - dimensional (level set) surface in $\mathbf{R}^{n+1}$.

3 points

Problem 3. Let now f from problem 2 have the form $f(t,r) = u(t) - r$ for a smooth function $u : \mathbf{R} \to \mathbf{R}$ with $u(t) > 0$ for $t \in \mathbf{R}$.

(a) In the case $n = 2$ determine all tangent vectors to S at $p = (u(t)\cos\theta, u(t)\sin\theta, t)$ for $t, \theta \in \mathbf{R}$.

(b) For general n show that at an arbitrary point $p = (x_1, x_2, \ldots, x_{n+1}) \in S$ one has $\nabla g(p) = (q, u'(x_{n+1}))$, where $q \in S^{n-1} = \{(x_1, \ldots, x_n) \in \mathbf{R}^n,\, x_1^2 + \ldots + x_n^2 = 1\}$ (u' denotes the derivative function of u.). Hence show that $n-1$ tangent vectors are of the form $v = (v_1, \ldots, v_n, 0)$ where $(v_1, \ldots, v_n) \in \mathbf{R}^n$ is a tangent vector to S^{n-1} in q. Determine a further vector in T_pS which is orthogonal to all those tangent vectors. This then gives a basis for T_pS.

4 points

To be submitted on Tuesday, 6. November 2018 at 10 a.m. in the lecture