Fachbereich Mathematik und Informatik
Freie Universität Berlin

Einführung in die Differentialgeometrie WS 2018/2019

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Sheet 2

Problem 1. Let $F : \mathbf{R}^2 \to \mathbf{R}^3$ be defined by $F(t,\theta) = (t\cos\theta, t\sin\theta, \theta)$. Show: $S = F(\mathbf{R}^2)$ is a parametrized 2- surface in $\mathbf{R}^3$. S is called the *helocoid*. Compute in particular the metric for S.

3 points

Problem 2. Let $S^n = \{x \in \mathbf{R}^{n+1}, |x|^2 = 1\}$ and $e_{n+1} = (0,\dots,0,1)$.
(a) Show: $S^n \backslash \{e_{n+1}\}$ is a parametrized n - surface with parametrization $F : \mathbf{R}^n \to \mathbf{R}^{n+1}$ given by

$$F(u) = \frac{1}{|u|^2+1}(2u, |u|^2 - 1),\ u \in \mathbf{R}^n.$$

Compute in particular the metric $(g_{ij}(u))$ für $u \in \mathbf{R}^n$.
(b) Determine the *stereographic projection* $F^{-1} : S^n \backslash \{e_{n+1}\} \to \mathbf{R}^n$ for the map F from part (a).

4 points

Problem 3. Let $b_1, \dots, b_n$ be linearly independent vectors in $\mathbf{R}^{n+1}$. Let M be the $n \times (n+1)$ - matrix with rows given by $b_1, \dots, b_n$. For $1 \le i \le n+1$ let M^i be the $n \times n$ - matrix obtained by deleting the i-th column of M and define $X_i = (-1)^{n+1+i} \det M^i$. The vector $X = (X_1, \dots X_{n+1})$ is often denoted by $b_1 \times \dots \times b_n$.
(a) Show: $X \neq 0$.
(b) Show: $X \cdot b_j = 0$ for $1 \le j \le n$.
(c) Show: The $(n+1) \times (n+1)$ - matrix with rows $b_1, \dots, b_n$ and X (in that order) has positive determinant.

3 points

To be submitted on Tuesday 13. November 2018 at 10 a.m. in the lecture