Fachbereich Mathematik und Informatik
Freie Universität Berlin

Einführung in die Differentialgeometrie WS 2018/2019

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Sheet 3

Problem 1. Let $\alpha : \mathbf{R} \to \mathbf{R}^2$ be defined by $\alpha(t) = (t, u(t))$ for a smooth function $u : \mathbf{R} \to \mathbf{R}$.

(a) Show: $C = \alpha(\mathbf{R})$ is a parametrized curve.

(b) Compute the curvature of C at the point $\alpha(t)$.

3 points

Problem 2. Let $F(t, \theta) = (t \cos\theta, t \sin\theta, \theta)$ for $(t, \theta) \in \mathbf{R}^2$ be the parametrization of the helicoid $S = F(\mathbf{R}^2)$ from problem 1, sheet 2.

(a) Determine the matix of the Weingarten map of F at the point $F(t, \theta)$ with respect to the basis vectors $\frac{\partial F}{\partial t}(t, \theta)$ and $\frac{\partial F}{\partial \theta}(t, \theta)$.

(b) Compute the Gauß curvature, the mean curvature as well as the principal curvatures of S at this point. **3 points**

Problem 3. Let $\alpha : I \to \mathbf{R}^2$ be given by $\alpha(t) = (x(t), y(t))$, $t \in I$ with $y(t) > 0$ and $x'^2(t) + y'^2(t) > 0$ for all $t \in I$. Let $F : I \times \mathbf{R} \to \mathbf{R}^3$ be the parametrization of the surface which one obtains by rotation of this curve about the x_1 - axis, i.e. $F(t, \theta) = (x(t), y(t) \cos\theta, y(t) \sin\theta)$ for $t \in I$ and $\theta \in \mathbf{R}$.

(a) Show: The Gauß curvature at the point $F(t, \theta)$ is given by

$$K(t, \theta) = \frac{x'(x''y' - x'y'')}{y(x'^2 + y'^2)^2}(t).$$

(b) Show: If $x'^2(t) + y'^2(t) = 1$ for all $t \in I$ then

$$K(t, \theta) = -\frac{y''}{y}(t).$$

(c) Let

$$x(t) = \int_0^t \sqrt{1 - e^{-2\tau}} d\tau, \qquad y(t) = e^{-t}$$

für $t > 0$. Show: $K(t, \theta) = -1$ for all (t, θ), $t > 0$. This surface is called the *pseudosphere*.

4 points

To be submitted on Tuesday 20. November 2018 at 10 a.m. in the lecture