Fachbereich Mathematik und Informatik
Freie Universität Berlin

Einführung in die Differentialgeometrie WS 2018/2019

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Sheet 7

Problem 1. Write down an expression for the components R_{ijkl} of the curvature tensor, which consists only of the components g_{ij} of the metric, of the Christoffelsymbols Γ^k_{ij} as well as the partial derivatives of the Christoffelsymbols.

2 points

Problem 2. Let M be an n - dimensional differentiable manifold with atlas $\{(x_\alpha, U_\alpha)\}$. Let $V \subset M$. We call V an *open subset* of M if for every α the sets $x_\alpha(V \cap U_\alpha)$ are open in $\mathbf{R}^n$. **(This is a definition!)**

(a) Show: The open subset of M defined in this way form a topology of M.

(b) Show: All chart mappings x_α are continuous with respect to this topology. (Note that this was not an assumption in our definition of a differentiable manifold.)

4 points

Problem 3. Let S^n be the unit sphere in $\mathbf{R}^{n+1}$. Let $N = e_{n+1}$ and $S = -e_{n+1}$ be the north pole and south pole of S^n, respectivlely. Show that there is a differentiable atlas $\{(x_N, U_N), (x_S, U_S)\}$ for S^n given by $U_N = S^n \backslash \{N\}$ and $U_S = S^n \backslash \{S\}$ as well as the associated stereographic projections $x_N : U_N \to \mathbf{R}^n$ und $x_S : U_S \to \mathbf{R}^n$ defined by

$$x_N(p_1, \ldots, p_{n+1}) = \frac{(p_1, \ldots, p_n)}{1 - p_{n+1}}$$

and

$$x_S(p_1, \ldots, p_{n+1}) = \frac{(p_1, \ldots, p_n)}{1 + p_{n+1}}.$$

4 points

To be returned on Monday 14. January 2019 at 12 noon (sharp) to Klaus Ecker's tutorial box