Polygons with total curvature smaller than 6π can bound only finitely many immersed minimal surfaces

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We shall sketch a partial proof of the assertion that a simple closed polygon $\Gamma \subset \mathbb{R}^3$ can bound only finitely many immersed minimal surfaces of disc-type if it meets two requirements: firstly it has to span only minimal surfaces without boundary branch points and secondly its total curvature, i.e. the sum of the exterior angles $\{\delta_l\}$ at its $N+3$ vertices, has to be smaller than 6π.
Here a disc-type minimal surface X is called *immersed* if there holds $\inf_B \mid DX \mid > 0$, where we denote by B the open unit disc $\{w = (u, v) \in \mathbb{R}^2 \mid \mid w \mid < 1\}$.

The considered statement should be compared to the following two achievements:
1) Nitsche proved in 1978 that a Jordan curve Γ of class $C^{4,\alpha}$ can bound only finitely many minimal surfaces, if it has the property to span only immersed minimal surfaces and if its total curvature does not exceed the value 6π.
2) After that Sauvigny proved in 1989 that a Jordan curve Γ of class $C^{4,\alpha}$ which is contained in the boundary of some convex compact set and in the open unit ball $B_1^3(0)$ can bound only finitely many immersed, small H-surfaces, if Γ meets the additional property

$$\frac{H^2\,(1+H)}{2\pi\,(1-H)}\mathcal{L}(\Gamma)^2 + \textit{total curvature}(\Gamma) \leq 6\pi.$$

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